Phys 410

Fall 2015

Homework #0

Due Thursday, 3 September, 2015

These are math skills that you will need for Phys 410. Review any concepts that present difficulty. Complete the following by hand (no assistance from computers!):

1. Use the Euler formula to expand $e^{i\theta}$ for real θ.
2. Given the three Cartesian unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$, calculate the following:
   a. $\hat{x} \times \hat{y}$
   b. $|\hat{x}|$
   c. $\hat{x} \cdot \hat{y}$
3. Given the vectors $\vec{r} = (r_x, r_y, r_z)$ and $\vec{s} = (s_x, s_y, s_z)$, calculate the cross product vector $\vec{r} \times \vec{s}$ in terms of its Cartesian components.
4. Find the eigenvalues and eigenvectors of this matrix: $\bar{\bar{A}} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$.
5. What is the determinant of $\bar{\bar{A}}$ and how is it related to the eigenvalues?
6. What is the trace of $\bar{\bar{A}}$ and how is it related to the eigenvalues?
7. Given a scalar function of position $\chi(\vec{r})$ (e.g. the temperature distribution on the surface of the earth), what can we say is always true about the curl of the gradient of χ?
8. Given a vector field $\vec{F} = k(x, 2y^2, 3z^3)$, where k is a constant, calculate its curl, $\nabla \times \vec{F}$. If F is a physical field (flow, force, etc.), what is the physical interpretation of $\nabla \times \vec{F}$?
9. Calculate the vector divergence of $\vec{F}$, namely $\vec{\nabla} \cdot \vec{F}$. What is the physical interpretation of this vector divergence?
10. What is the general solution to the second-order linear differential equation $\ddot{x} = -\omega^2 x$, where ω is a real positive number?
11. What is the general solution to the second-order linear differential equation $\ddot{x} = +k^2 x$, where k is a real positive number?
12. Given $ln(y) = b\ ln(x)$, where b is a constant, find y as a function of x, $y(x)$.
13. Evaluate the integral $I = \int_{-2}^{3} 5x\, dx$.
14. Expand $y(x) = \ln(1 + x)$ to second order for $x \ll 1$. Write the series expansion for $y(x) = \frac{1}{1-x}$ valid for $-1 < x < 1$.
15. Consider a function in 3-dimensional space, $\varphi(\vec{r}) = \varphi(x, y, z)$. Defining a differential displacement as $d\vec{r} = dx\,\hat{\imath} + dy\,\hat{\jmath} + dz\,\hat{k}$, where dx, dy, and dz are independent step sizes, calculate explicitly the change in the function φ (in Cartesian coordinates) that comes about from this displacement: $d\varphi = \vec{\nabla}\varphi \cdot d\vec{r}$. [for review, see section 4.3 of Taylor]
16. For a scalar function of position $\varphi(\vec{r})$, explain the physical meaning of $\vec{\nabla}\varphi(\vec{r})$.
17. State the first fundamental theorem of calculus. What is an anti-derivative?

18. Suppose we have a function representing a physical quantity $y(x, p, t)$ where x is position, p is momentum and t is time, and determine that a differential change in this function in differential time dt is given by $dy = 12\,dx - 34\pi\,dp + 27000\,dx\,dp$. Now calculate the rate at which the function changes, namely $\lim_{dt\to 0} dy/dt$. What can we conclude about the $dx\,dp$ term?
19. Write down the differential volume element d^3r in spherical coordinates. Use the figure below for definition of the spherical coordinates.